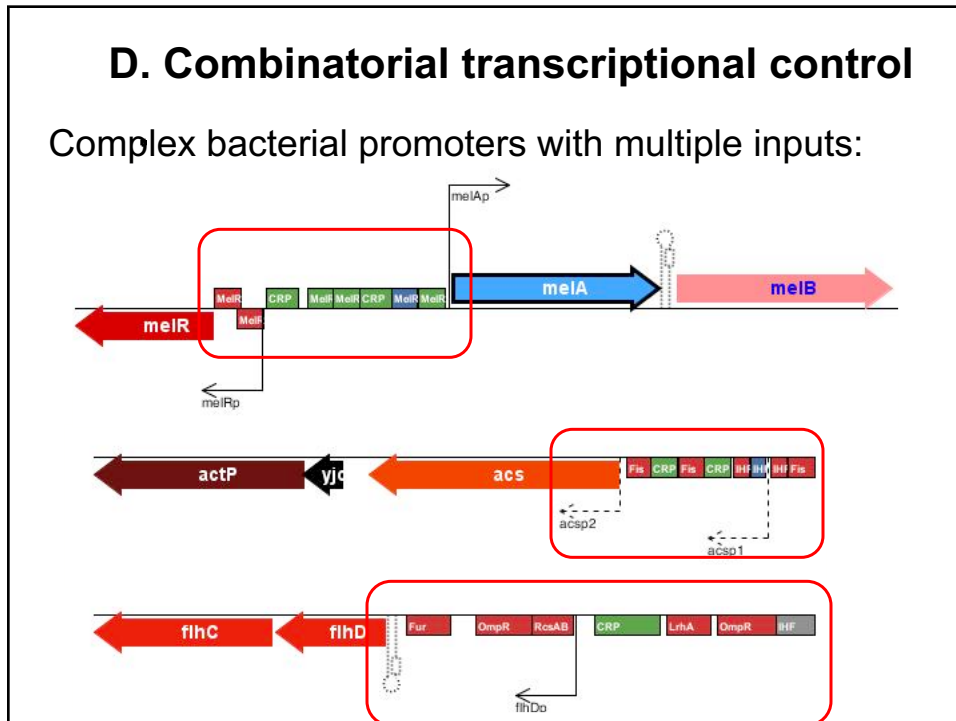


D. Combinatorial transcriptional control

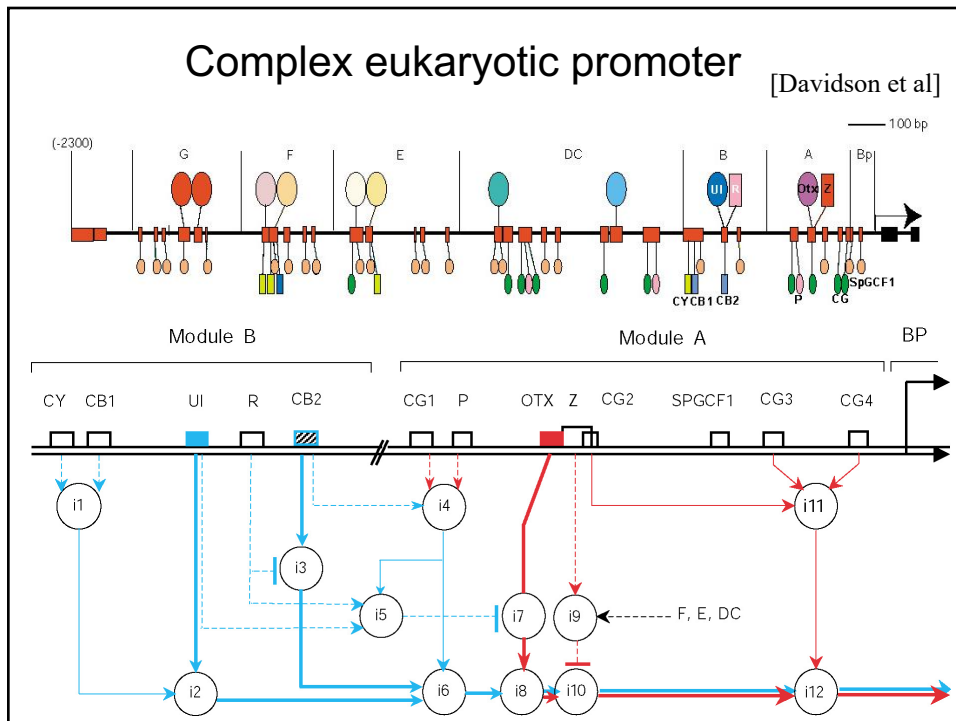
Complex bacterial promoters with multiple inputs:



1

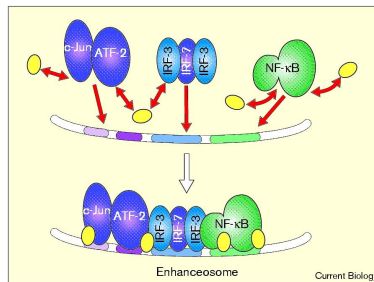
Complex eukaryotic promoter

[Davidson et al]



2

Mechanisms of complex transcriptional control?



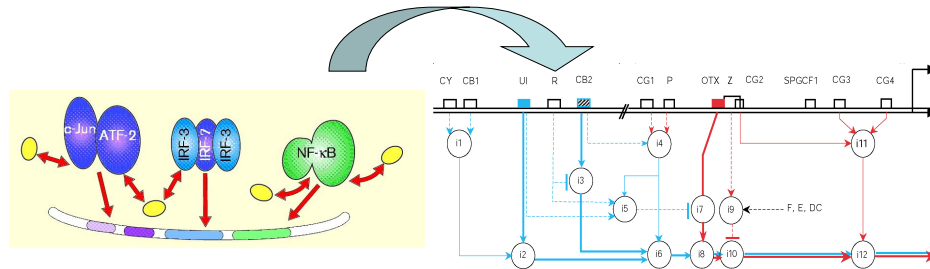
- specific, complex protein-protein interaction
 ... but different TFs can work together to implement different functions
 → **combinatorial control favors simpler, less specific interaction**
- alternative: **regulated recruitment** [Ptashne & Gann '97]
 - simple, glue-like interaction between TFs/RNAP
 - arrange DNA binding sites/strengths to accomplish **desired** control functions
 - ... but how to implement? and what are the limitations?

3

Theory of Combinatorial Transcription Control

[Buchler et al, PNAS '03]

Statistical Mechanics



4

Theory of Combinatorial Transcription Control

[Buchler et al, PNAS '03]

quantitative description: via changes in **regulatory sequences alone**

- occupation of site j : $\sigma_j = \{0,1\}$
- TF-operator interaction: $K_j = \exp(-\beta\Delta G_j) = 1 \sim 1000 \text{ nM}$
- TF-TF and TF-RNAP interaction: $\omega_{ij} = \exp(-\beta E_{ij}) = \{0, 1, 10 \sim 100\}$
 - **regulated recruitment** [M. Ptashne]
 - long-distance interaction possible via DNA looping
- **promoter activity** \sim **equilibrium** promoter occupation prob $\mathcal{P}([TF_j]) \approx \langle \sigma_p \rangle$ [Shea & Ackers, '85]

→ thermodynamic model
$$Z = \sum_{\{\sigma_i\}} \prod_{j=1}^N \left([TF_j] / K_j \right)^{\sigma_j} \cdot \prod_{i < j} \omega_{i,j}^{\sigma_i \sigma_j}$$

→ **programmable molecular Boltzmann machine!**

→ What kind of control functions $\mathcal{P}([TF_j])$ are implementable via the appropriate choices of $\{K_j, \omega_j\}$?

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1. non-interacting

simple activation:

$$W_{\text{off}} = 1 + \frac{[A]}{K_A}$$

$$W_{\text{on}} = \frac{[RNAP]}{K_p} + \omega_{A-p} \cdot \frac{[A]}{K_A} \cdot \frac{[RNAP]}{K_p}$$

$$\mathcal{P} \approx \frac{W_{\text{on}}}{W_{\text{off}}} = \frac{[RNAP]}{K_p} \cdot \frac{1 + \omega_{A-p} [A] / K_A}{1 + [A] / K_A}$$

(for typical weak promoters)

simple repression:

$$\mathcal{P} \approx \frac{[RNAP]}{K_p} \cdot \frac{1}{1 + [R] / K_R}$$

co-regulation multiplicative

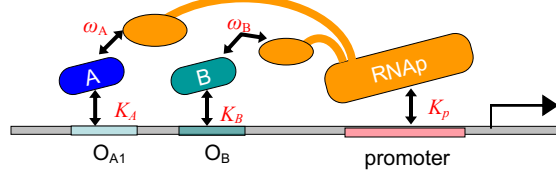
$$\mathcal{P} \propto \frac{1 + \omega_{A-p} [A] / K_A}{1 + [A] / K_A} \cdot \frac{1}{1 + [R] / K_R}$$

[example: lac promoter (details later)]

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2. Synergistic activation

RNAP can simultaneously contact two TFs (e.g., Crp and Fnr)



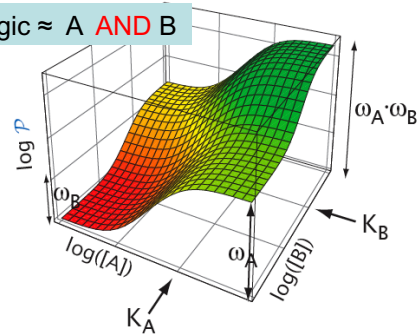
statistical weight W for each configuration $\{\sigma_A, \sigma_B, \sigma_p\}$, with $q_X = [X]/K_X$

$$\begin{aligned}
 W_{off} & \begin{cases} W(0,0,0) = 1 \\ W(1,0,0) = q_A \\ W(0,1,0) = q_B \\ W(1,1,0) = q_A \cdot q_B \end{cases} \\
 W_{on} & \begin{cases} W(0,0,1) = q_p \\ W(1,0,1) = \omega_A \cdot q_A \cdot q_p \\ W(0,1,1) = \omega_B \cdot q_B \cdot q_p \\ W(1,1,1) = \omega_3 \cdot q_A \cdot q_B \cdot q_p \end{cases}
 \end{aligned}$$

consider $\omega_3 \approx \omega_A \cdot \omega_B$

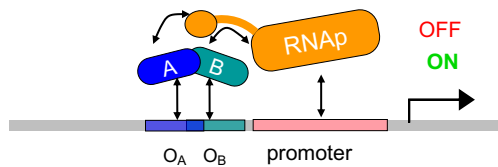
$$\mathcal{P}([A],[B]) \approx \frac{W_{on}}{W_{off}} \approx q_p \frac{1 + \omega_A q_A}{1 + q_A} \cdot \frac{1 + \omega_B q_B}{1 + q_B}$$

logic \approx A AND B

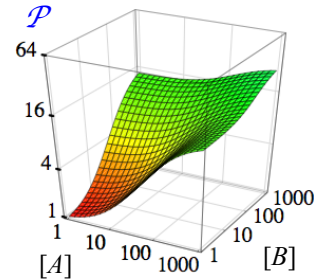


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3. Competitive (or "independent") activation

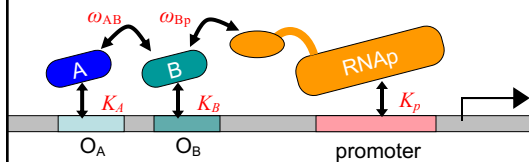


$$\mathcal{P}([A],[B]) \approx q_p \frac{1 + \omega_A q_A + \omega_B q_B}{1 + q_A + q_B}$$

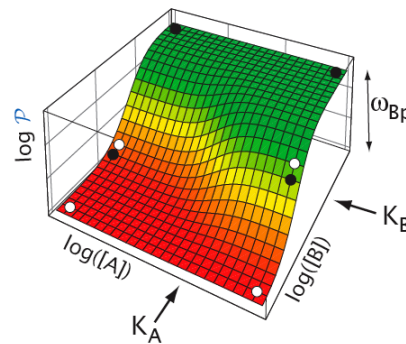


logic \approx A OR B

4. Cooperative activation



could work as AND or OR by choosing K_A, K_B



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5. Cooperative repression

note: direct interaction between A and B not necessary (cf "collaborative competition")

6. Competitive repression

logic \approx NAND

logic \approx NOR

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More complex control function, e.g., XOR ?

A/B	AND	OR	NAND	XOR	EQ
lo/lo	OFF	OFF	ON	OFF	ON
lo/hi	OFF	ON	ON	ON	OFF
hi/lo	OFF	ON	ON	ON	OFF
hi/hi	ON	ON	OFF	OFF	ON

cannot be implemented by overlapping A and B sites

[cf: linear perceptron (Minsky '69)]

XOR(A,B) = (A OR B) AND NOT(A AND B)

Gene cascade

problems:

- need a gene for each intermediate result
- multiple rounds of gene expression: noise + delay
- synchronization difficult
- amplification nontrivial

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More complex control function, e.g., **XOR** ?

A/B	AND	OR	NAND	XOR	EQ
lo/lo	OFF	OFF	ON	OFF	ON
lo/hi	OFF	ON	ON	ON	OFF
hi/lo	OFF	ON	ON	ON	OFF
hi/hi	ON	ON	OFF	OFF	ON

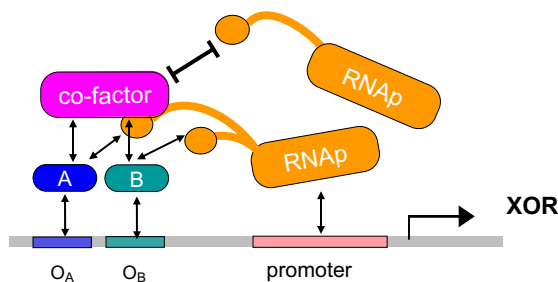
cannot be implemented by overlapping A and B sites



[cf: linear perceptron (Minsky '69)]

$$\text{XOR}(A,B) = (A \text{ OR } B) \text{ AND NOT}(A \text{ AND } B)$$

Allosteric or co-factor mediated



problem:

- need dedicated component
- lose combinatorial control e.g., can't implement **AND** elsewhere

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More complex control function, e.g., **XOR** ?

A/B	AND	OR	NAND	XOR	EQ
lo/lo	OFF	OFF	ON	OFF	ON
lo/hi	OFF	ON	ON	ON	OFF
hi/lo	OFF	ON	ON	ON	OFF
hi/hi	ON	ON	OFF	OFF	ON

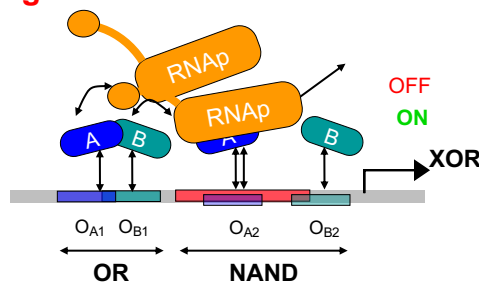
cannot be implemented by overlapping A and B sites



[cf: linear perceptron (Minsky '69)]

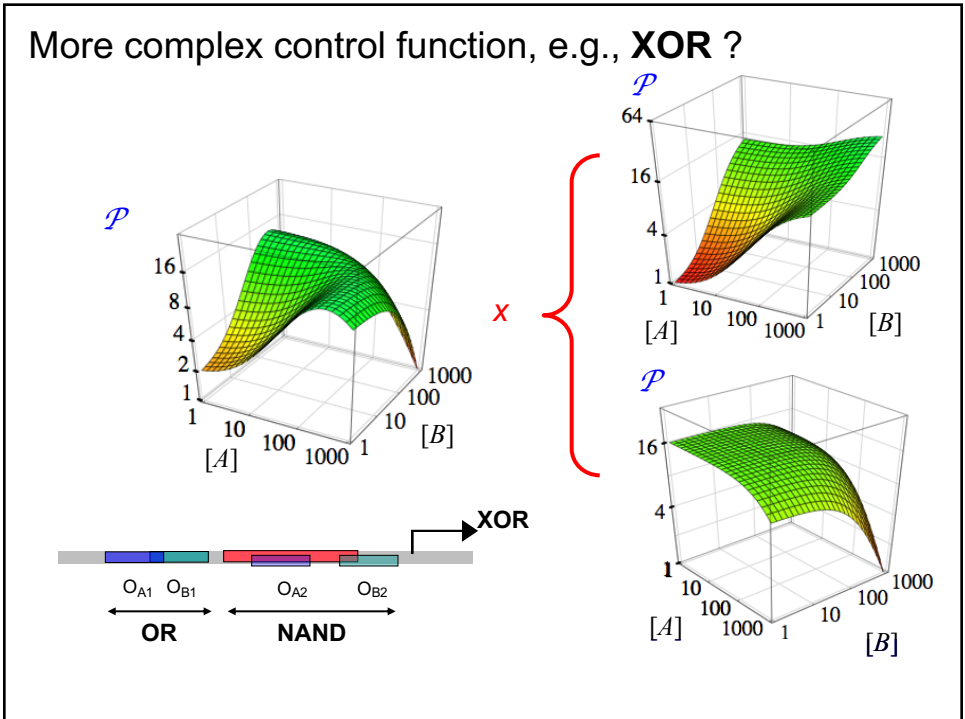
$$\text{XOR}(A,B) = (A \text{ OR } B) \text{ AND NOT}(A \text{ AND } B)$$

Regulated recruitment

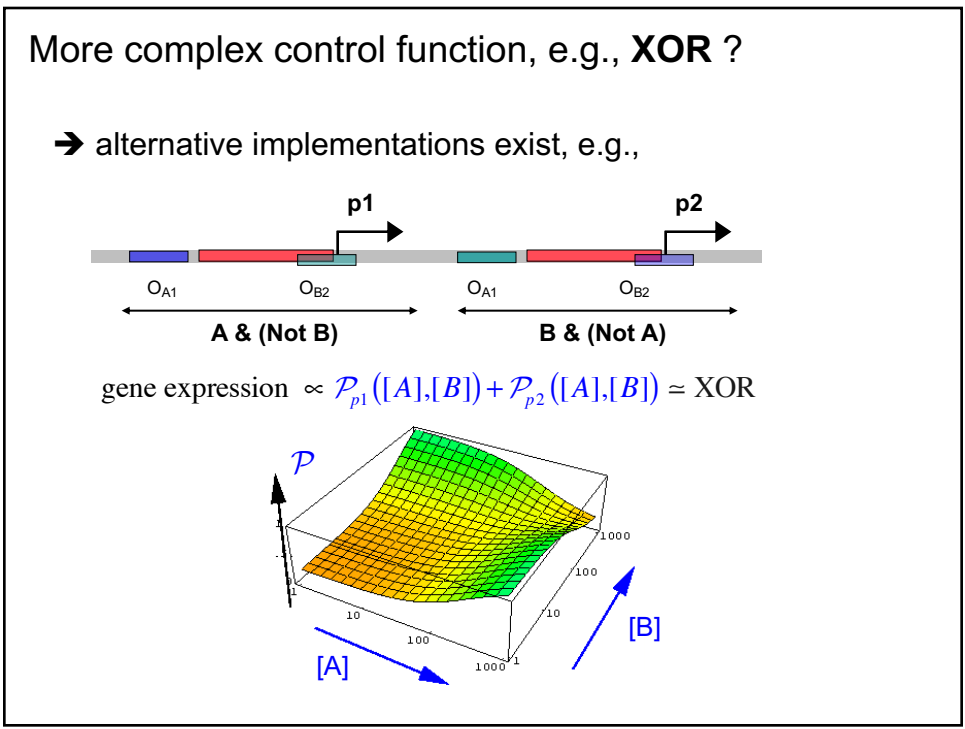


- integrates **OR** and **NAND** into a single regulatory region
- no need for special proteins
- modular and evolvable

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XOR promoter in bacteria?

Crp: senses carbon shortage
 NtrC~P: senses nitrogen shortage
 function of *glnHPQ* gene product: Glutamine transporter

Glutamine (C₅H₁₀N₂O₃)

use as carbon source under carbon shortage

use as nitrogen source under nitrogen shortage

→ need quantitative characterization of such promoters
 → design of synthetic promoters using exogenous regulators

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EQ gate?

A/B	EQ
lo/lo	ON
lo/hi	OFF
hi/lo	OFF
hi/hi	ON

- strong promoter
- need **multiple** ways of repression
 A (low) B (high) or A (high) B (low)

→ competitive promoter binding awkward due to **limited promoter size** (dead end)

→ possible solution: **interaction at a distance**
 e.g., DNA-looping via dimers (AraC, GalR, MelR,...)

heterodimers: [A. Hochschild et al]

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E.g., **distal repression** by heterodimer pair R and R'

allow control by TF **A**

site A site R' (weak) site R promoter (weak)

Implementation of **EQ gate**: **Effective cascade w/o new genes!**

A1 R' B1 B2 R' A2 R default ON

A & NOT B B & NOT A

A1 R' B1 B2 R' A2 R default OFF

NOT A & NOT B A & B

[A] [B]

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Generalized control architecture w/ multiple TF's

C R' B C R' A D B R

C AND B C AND A D OR B

NOT

Output = [...OR...OR...] & NOT[...&...&...] & NOT[...&...&...]

phenotype: **dominant repression** **Conjunctive Normal Form**

C R' B C R' A D B R

C AND B C AND A D AND B

NOT

Output = [...&...&...] OR [...&...&...] OR [...&...&...]

phenotype: **enhancer autonomy** **Disjunctive Normal Form**

→ all logic functions reducible to **minimal CNF or DNF**

→ **schemata** for constructing arbitrary regulatory logic functions (**programmable** molecular computer!)

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Molecular computer ↔ **“Neural network”**

TF concentration (n)	input
TF binding sites (j)	neurons
promoter activity (P)	output
binding strength (K_j)	firing threshold
TF-TF interaction (ω_{ij})	synapse

➔ single node of GRN is already a **network!**

- symmetric interaction
- “hidden units” (cofactors)

} **Molecular Boltzmann machine!**

“learning” ↔ **evolution of regulatory sequences**

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Summarize:

A large variety of control functions $\mathcal{P}([TF_j])$ may be implementable via appropriate choices of $\{K_j, \omega_{ij}\}$, i.e., via **regulatory sequences alone** = **programmable molecular computer**

- ➔ **synthesize** “desired” transcriptional “logic gates”
- ➔ **“breed”** regulatory sequences to implement desired control functions

Potential application: **cell-specific** gene expression profiling

	cell A	cell B	cell C	cell X
gene 1	+	-	+	-
gene 2	-	+	+	+
gene 3	-	-	+	+
...				

- ➔ cell type discrimination: use **multiple traits**
- ➔ **cell X revealed** by a reporter gene driven by designed regulatory sequences
- ➔ **cell X eliminated** by promoter activating apoptosis
- ➔ targeted delivery not required (~ **smart bomb!**)

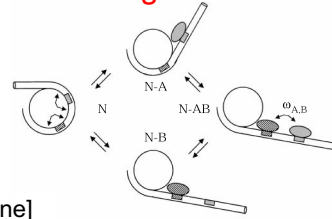
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Ingredients for complex transcription control

- programmable protein-DNA interaction
- weak, glue-like interaction between nearby proteins
- **long-distance activation/repression**
- **insulation of gene regulatory control**

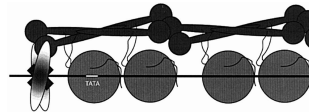
Eukaryotes: **formalism as phenomenological model**

- generic cooperative interaction mediated by **nucleosomes**
[Polach & Widom, '96]
- physical attraction not necessary



- **short-range “quenching”** [Arnosti, Levine]

- distal repression via recruitment of **chromatin modification** agents



- **insulating elements**: crucial for minimizing cross talk

similar ingredients but superior implementation platform