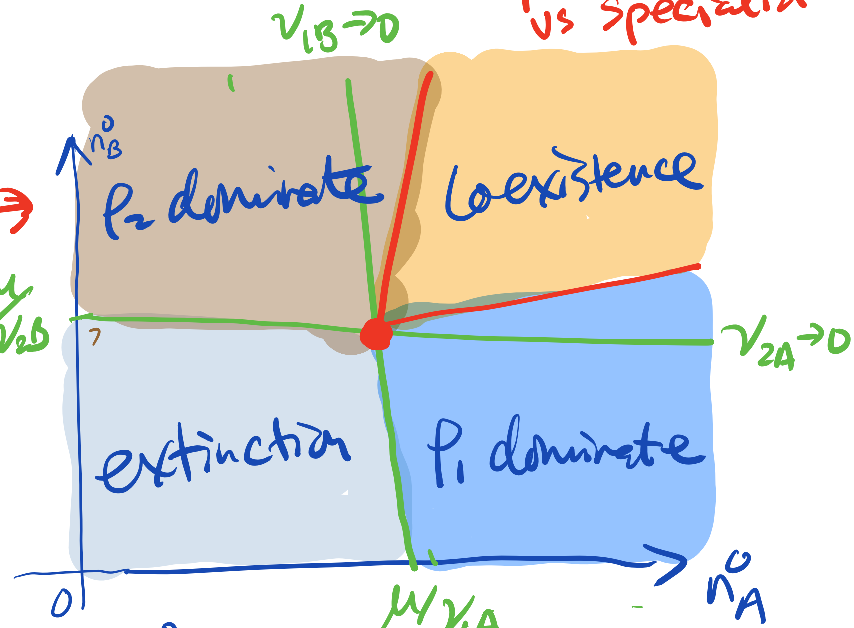
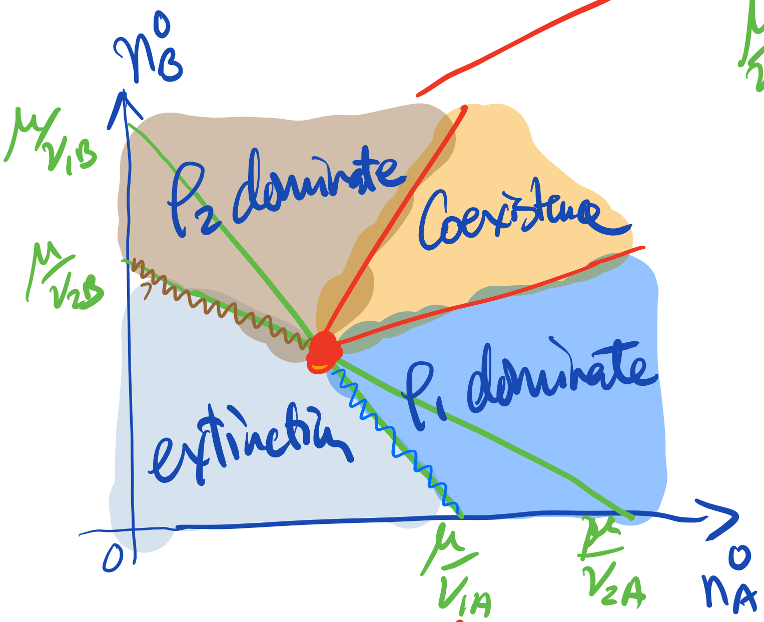
B3. phase diagram and feasibility space

a) Ecological phase diagram (for 2 species)

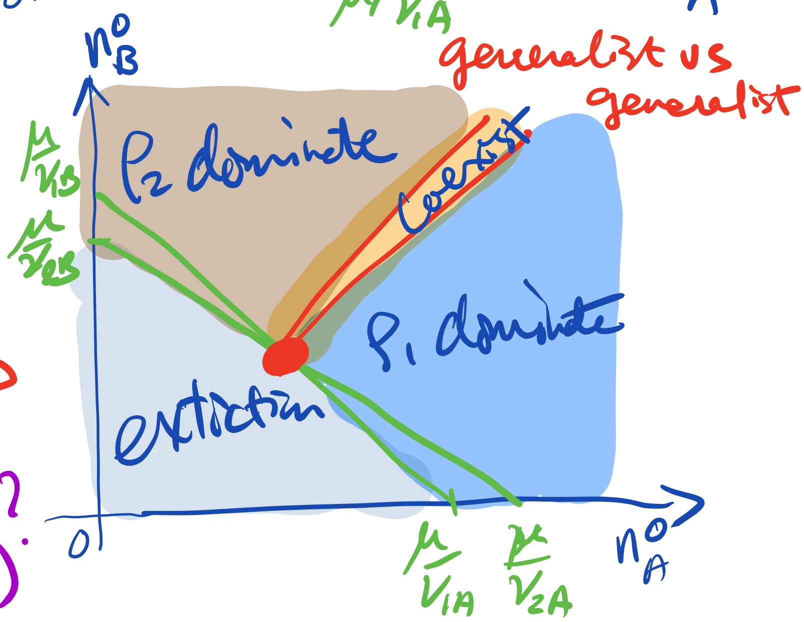
Strong orthogonal nutrient pref
broad coexist. regime

specialist vs specialist





generalist vs generalist



Similar nut. pref.
narrow coexist regime

$\Rightarrow$ phenotypical landscape for fixed environment (n_A^0, n_B^0)?

* Algebraic analysis:

$$\begin{bmatrix}\rho_1^*\\ \rho_2^*\end{bmatrix} = \begin{bmatrix} \nu_{2B}\dfrac{n_A^0-n_A^*}{\nu_{2B}-\nu_{1B}}Y_A - \nu_{2A}\dfrac{n_B^0-n_B^*}{\nu_{1A}-\nu_{2A}}Y_B \\ -\nu_{1B}\dfrac{n_A^0-n_A^*}{\nu_{2B}-\nu_{1B}}Y_A + \nu_{1A}\dfrac{n_B^0-n_B^*}{\nu_{1A}-\nu_{2A}}Y_B\end{bmatrix} = \begin{bmatrix}\dfrac{\nu_{2B}}{\nu_{2B}-\nu_{1B}}\, j_A/\mu - \dfrac{\nu_{2A}}{\nu_{1A}-\nu_{2A}}\, j_B/\mu \\ \dfrac{-\nu_{1B}}{\nu_{2B}-\nu_{1B}}\, j_A/\mu + \dfrac{\nu_{1A}}{\nu_{1A}-\nu_{2A}}\, j_A/\mu\end{bmatrix}$$

Where $j_\alpha^* = \mu(n_\alpha^0 - n_\alpha^*)Y_\alpha$ = flux of nutrient α assimilated

Note: $\rho_1^* + \rho_2^* = (j_A^* + j_B^*)/\mu$ (mass conservation) let $\psi_i \equiv \rho_i^* / (\rho_1^* + \rho_2^*)$; frac. abundance of sp. i

$f_\alpha^* \equiv \dot{j}_\alpha^* / (\dot{j}_A^* + \dot{j}_B^*)$; frac. assim. flux for nutrient α

then $\psi_1 = f_A^* \dfrac{\nu_{2B}}{\nu_{2B} - \nu_{1B}} - f_B^* \dfrac{\nu_{2A}}{\nu_{1A} - \nu_{2A}}$ from mass conservation

In limit $\mu \to 0$ (emphasizes effect of competition)

$$n_\alpha^* \propto \mu \ll n_\alpha^0$$

then $\dot{j}_\alpha^* = \mu (n_\alpha^0 - n_\alpha^*) Y_\alpha \simeq \mu n_\alpha^0 Y_\alpha$ ← environmental parameter.

$$\psi_1 = f_A \frac{\nu_{2B}}{\nu_{2B} - \nu_{1B}} - f_B \frac{\nu_{2A}}{\nu_{1A} - \nu_{2A}};$$

where $f_\alpha = \dfrac{\dot{j}_\alpha}{\dot{j}_A + \dot{j}_B} = \dfrac{n_\alpha^0 Y_\alpha}{n_A^0 Y_A + n_B^0 Y_B}$ involves only env. parameters

let $m_\alpha \equiv \dfrac{\nu_{1\alpha}}{\nu_{2\alpha}}$ be uptake preference of species 1 for nutrient α rel. to species 2 for α

note: different from $m_1 = \dfrac{\nu_{1A}/Y_A}{\nu_{1B}/Y_B}$ used earlier

$$\rightarrow \psi_1 = \frac{f_A}{1 - m_B} - \frac{f_B}{m_A - 1}$$

- condition for coexistence: $1 > \psi_1 > 0$

- if $m_A > 1$, $m_B > 1$, $\psi_1 < 0$ } no coexistence
- if $m_A < 1$, $m_B < 1$, $\psi_1 > 1$
- for $m_A > 1 > m_B$ or $m_A < 1 < m_B$

$\psi_1 > 0 \quad \dfrac{f_A}{1 - m_B} > \dfrac{f_B}{m_A - 1}$,

$f_A m_A - f_A > f_B - m_B f_B \rightarrow \boxed{f_A m_A + f_B m_B > 1}$

$\psi_1 < 1$. $\frac{f_A}{1-m_B} - \frac{f_B}{m_A-1} < 1$

$m_A m_B < m_A f_B + m_B f_A \rightarrow \boxed{\frac{f_A}{m_A} + \frac{f_B}{m_B} > 1}$

→ Ecological phase diagram: range of f_A given (m_A, m_B)

$m_A > 1 > m_B$
or $m_B > 1 > m_A$

$f_A m_A + (1-f_A) m_B > 1$

if $m_A > m_B$ then $f_A > \frac{1-m_B}{m_A-m_B}$

if $m_A < m_B$, then $f_A < \frac{m_B-1}{m_B-m_A}$

$f_A m_A^{-1} + (1-f_A) m_B^{-1} > 1$

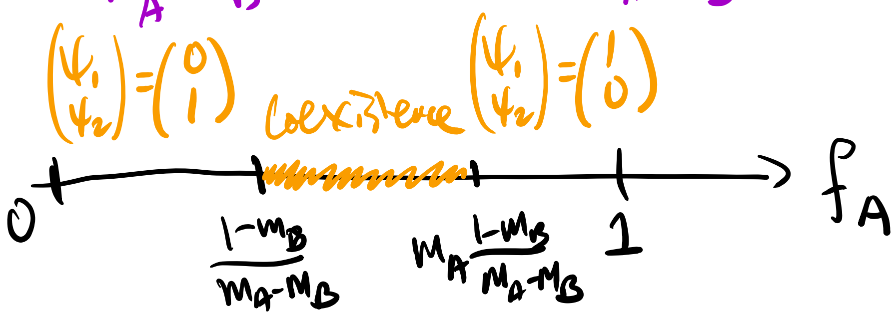
if $m_B > m_A$, then $f_A > \frac{m_A(m_B-1)}{m_B-m_A}$

if $m_A > m_B$, then $f_A < \frac{m_A(1-m_B)}{m_A-m_B}$

for $m_B < 1 < m_A$: $\frac{1-m_B}{m_A-m_B} < f_A < m_A \frac{1-m_B}{m_A-m_B}$

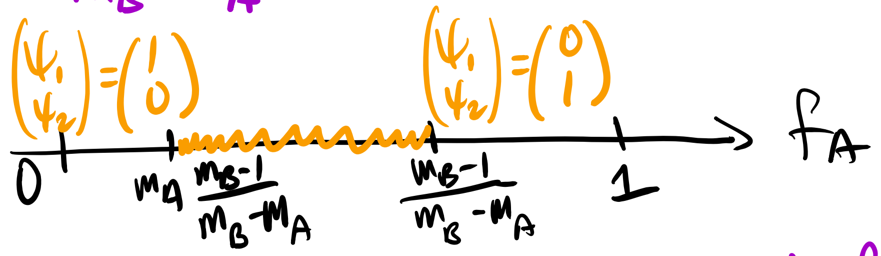
V_{1B}/V_{2B} $\quad$ V_{1A}/V_{2A}

Sp1 specializes in A
Sp2 specializes in B



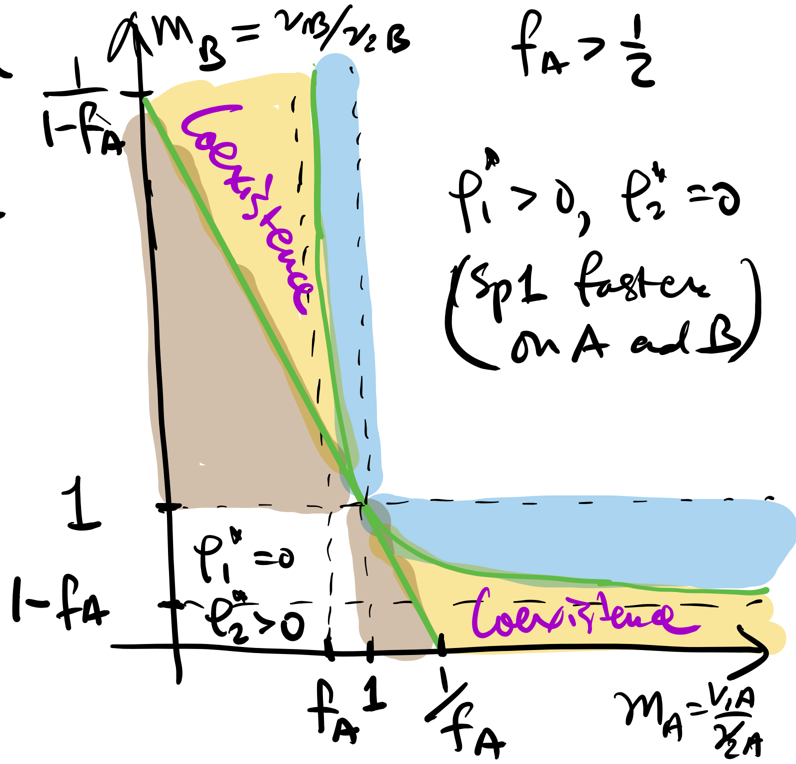
for $m_A < 1 < m_B$: $\frac{m_A(m_B-1)}{m_B-m_A} < f_A < \frac{m_B-1}{m_B-m_A}$

Sp1 specializes in B
Sp2 specializes in A



⇒ Coexistence occurs for intermediate range of f_A
but meaning of condition obscure (see later)

b) phenotypical phase diagram for coexistence for <u>fixed</u> f_A



$\psi_1 > 0$: $m_A f_A + m_B f_B > 1$

$\psi_1 < 1$: $\frac{f_A}{m_A} + \frac{f_B}{m_B} > 1$

- Conditions favorable for coexistence:
  large m_A, small m_B or vice versa
  = niche specialization

- For a given f_A, species can change genetic parameter (m_A, m_B) to drive the other species to extinction! Thus, phenotypical phase diagram has element of "fitness landscape" (but $\psi \neq$ fitness)

  → challenge: f_A is variable

- however, trivial effect from overall scale of $\nu_{i\alpha}$: if $\nu_{1\alpha} > \nu_{2\alpha}$, $\rho_2 \to 0$
  $\nu_{1\alpha} < \nu_{2\alpha}$, $\rho_1 \to 0$

  → need to "fix" the scale of $\nu_{i\alpha}$